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TECHNICAL REPORT ARCCB-TR-99009

BAUSCHINGER EFFECT DESIGN PROCEDURES FOR AUTOFRETTAGED TUBES INCLUDING MATERIAL REMOVAL AND SACHS' METHOD

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MAY 1999

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
May 1999

3. REPORT TYPE AND DATES COVERED
Final

4. TITLE AND SUBTITLE
BAUSCHINGER EFFECT DESIGN PROCEDURES FOR AUTOFRETTAGED TUBES INCLUDING MATERIAL REMOVAL AND SACHS' METHOD

5. FUNDING NUMBERS
AMCMS No. 6226.24.H181.0

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
U.S. Army ARDEC
Benet Laboratories, AMSTA-AR-CCB-O
Watervliet, NY 12189-4050

8. PERFORMING ORGANIZATION REPORT NUMBER
ARCCB-TR-99009

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army ARDEC
Close Combat Armaments Center
Picatinny Arsenal, NJ 07806-5000

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
To be presented at the ASME Pressure Vessels and Piping Conference, Boston, MA, 1-5 August 1999.
To be published in proceedings of the conference.

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Autofrettage is used to introduce advantageous residual stresses into pressure vessels and to enhance their fatigue lifetimes. The Bauschinger effect serves to reduce the yield strength in compression as a result of prior tensile plastic overload and can produce lower compressive residual hoop stresses near the bore than are predicted by 'ideal' autofrettage solutions (elastic/perfectly plastic without Bauschinger effect). A complete analysis procedure is presented which encompasses representation of elastic-plastic uniaxial loading material behavior and of reverse-loading material behavior as a function of plastic strain during loading. Such data are then combined with some yield criterion to accurately predict elastic-plastic residual stress fields within an autofrettaged thick cylinder. Pressure for subsequent re-yielding of the tube is calculated. The numerical procedure is further used to determine residual stress fields after removal of material from inside diameter (ID) and/or outside diameter (OD), including the effects of any further plasticity. A specific material removal sequence is recommended. It is shown that Sachs' experimental method, which involves removing material from the ID, may very significantly overestimate autofrettage residual stresses near the bore. Stress ranges and stress intensity factors for cracks within such stress fields are calculated together with the associated fatigue lifetimes as such cracks propagate under cyclic pressurization. The loss of fatigue lifetime resulting from the Bauschinger effect is shown to be extremely significant.

14. SUBJECT TERMS
Bauschinger Effect, Autofrettage, Gun Tubes, Sachs' Method, Residual Stresses, Fatigue Life, Pressure Vessels

15. NUMBER OF PAGES
11

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

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ABSTRACT

Autofrettage is used to introduce advantageous residual stresses into pressure vessels and to enhance their fatigue lifetimes. The Bauschinger effect serves to reduce the yield strength in compression as a result of prior tensile plastic overload and can produce lower compressive residual hoop stresses near the bore than are predicted by 'ideal' autofrettage solutions (elastic/perfectly plastic without Bauschinger effect).

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Stress ranges and stress intensity factors for cracks within such stress fields are calculated together with the associated fatigue lifetimes as such cracks propagate under cyclic pressurization. The loss of fatigue lifetime resulting from the Bauschinger effect is shown to be extremely significant.

NOMENCLATURE

E	Elastic modulus
E_{eff}	Effective elastic modulus
L	Indicates peak of autofrettage loading
U	After autofrettage unloading
a, a', b, b', c, d, d'	Radii defined in Figure 2
p	Pressure
p^*	Pressure at peak of Autofrettage
p^{**}	Pressure for re-yielding
r	Radius
ε^p	Plastic strain
ε_c	Compressive strain
σ_c	Compressive stress
σ_Y	Uniaxial yield stress
σ_r	Radial stress
σ_θ	Hoop stress
σ_θ^A	Bore hoop residual stress after autofrettage
ν	Poisson's ratio
ν_{eff}	Effective Poisson's ratio

INTRODUCTION

Autofrettage is used to introduce advantageous residual stresses into pressure vessels and to enhance their fatigue lifetimes. For many years workers have acknowledged the probable influence of the *Bauschinger effect,* Bauschinger (1881), which serves to reduce the yield strength in compression as a result of prior tensile plastic overload. Chakrabarty (1987) provides some review of the microstructural causes.

The reduction of compressive yield strength within the yielded zone of an autofrettaged tube is of importance because, on removal of the autofrettage pressure, the region near the bore experiences high values of compressive hoop stress, approaching the normal tensile yield strength of the material if the unloading is totally elastic. If the combination of stresses exceeds some yield criterion the tube will re-yield from the bore thus losing much of the potential benefit of autofrettage.

There have been numerous papers, spanning almost two decades, relating to the impact of the Bauschinger effect upon residual stresses and associated fatigue lifetimes in autofrettaged thick cylinders. In

briefest summary these papers indicate that a series of effects needs to be recognized and tackled sequentially. These are:

a. Determination of elastic-plastic uniaxial loading material behavior

b. Determination of elastic-plastic uniaxial reverse-loading material behavior as a function of plastic strain during loading

c. Use of (a) and (b), in combination with some yield criterion, to predict elastic-plastic residual stress fields within an autofrettaged thick cylinder

d. Determination of residual stress fields after material removal from inside diameter (ID) and/or outside diameter (OD), including the effects of any further plasticity

e. Prediction of the pressure for subsequent re-yielding of the tube

d. Determination of stress intensity factors for cracks within such stress fields

e. Calculation of fatigue lifetimes as such cracks propagate under fatigue loading

The purpose of this paper is to propose and demonstrate a complete design procedure which encompasses all of the above elements. In addition it is possible to expand upon the question of material removal in order to critically examine a standard experimental procedure based upon material removal (Sachs' method) which has been used extensively to determine residual stresses in autofrettaged tubes.

Since the scope of this paper is extremely wide, detail of formulation is minimized and the reader may therefore need to refer in some detail to source material.

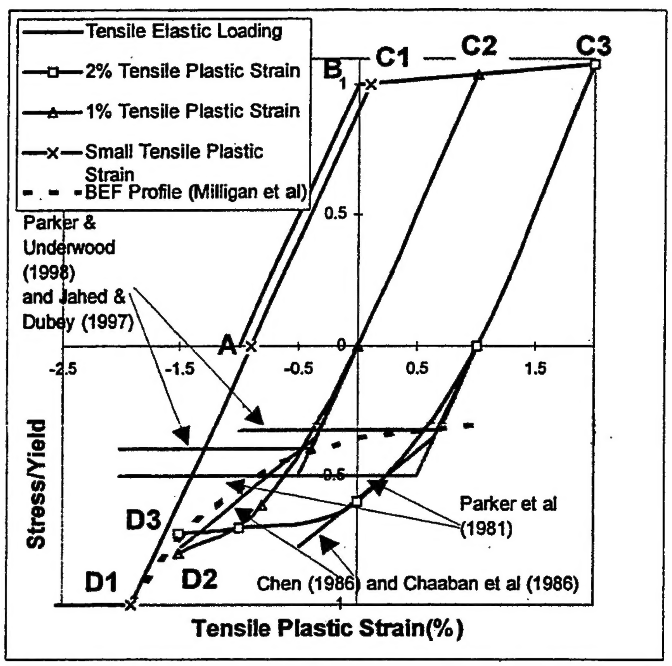

Figure 1 : Schematic Representation of Bauschinger Effect and Bi-linear Models

THE BAUSCHINGER EFFECT - UNIAXIAL STRESS-STRAIN PROFILES

Figure 1 shows a schematic of the uniaxial loading and reverse loading of a typical gun steel. The important features of the behavior are:

a. (A-B) A near-linear $\sigma-\varepsilon$ relationship during elastic loading of slope E (Young's modulus)

b. (B-C1-C2-C3) a linear or non-linear $\sigma-\varepsilon$ behavior during loading beyond the elastic limit, B.

c. (C1-D1, C2-D2, C3-D3) a linear $\sigma-\varepsilon$ behavior during reversal of the tensile load followed by an increasingly non-linear behavior as the loading becomes more compressive.

The Bauschinger effect factor (BEF) is the ratio of magnitude of true yield strength in compression to true yield strength in tension. BEF is clearly a function of prior plastic strain (B-Cn, n = 1, 2, 3 ....).

Milligan et al (1966) have characterized the Bauschinger effect in certain steels in the form of BEF for a range of offsets (0.05%, 0.10%, 0.20% and 0.25%). Schematically this data plots as a series of hatched lines, each relating to a different offset value, one of which is illustrated schematically in Figure 1. They vary from unity at zero plastic strain, drop rapidly with increasing plastic strain and saturate at around 2% plastic strain, being effectively constant thereafter.

Kendall (1998) has used the Milligan et al (1966) BEF data for the full range of offsets to reconstruct and fit the complete loading-reversed loading profile as a function of the *percentage* plastic strain during loading, ε^p. This fit, in a form which extends the validity of that in Kendall (1998), is given by:

$$\sigma_c/\sigma_Y = 0.064113 + 0.34816\exp(-12.871\varepsilon^p)$$
$$+[1.1619 + 0.5975\exp(-2.175\varepsilon^p)]\varepsilon_c$$
$$-[0.54959 + 0.74913\exp(-9.6376\varepsilon^p)]\varepsilon_c^2 \quad (1)$$

If σ_c exceeds the elastic value then $\sigma_c = E\varepsilon_c$. σ_c and ε_c are compressive stress and strain respectively during reversed loading and σ_Y is the uniaxial yield strength of the material.

AUTOFRETTAGE RESIDUAL STRESSES - PRIOR WORK

There have been several attempts to model the full 'surface' of the type represented by equation (1); Parker et al (1981), Chen (1986), Chaaban et al (1986), Parker and Underwood (1998), Megahed and Abbas (1991) and Jahed and Dubey (1997). With the exception of Megahed and Abbas (1991), all of these models employ a bi-linear representation of the unloading phase, Cn-Dn (n=1, 2, 3 ...), as illustrated schematically in Figure 1. The Chen (1986) and Chaaban et al (1986) models were subtly different in that the former employs a tangent-like slope of 0.3E to represent the linear compressive 'strain hardening' phase whereas the latter forces the final line through a point having coordinates of (-{total tensile strain + elastic tensile strain}, {-yield strength}) using coordinates of Figure 1.

Megahed and Abbas (1991), do attempt to follow a full non-linear unloading profile which is a function of ε^p; unfortunately the method is not usable because the analysis requires a specific analytical form

for Cn-Dn(ε^p) and this required form is physically unacceptable for a range of materials, in particular martensitic steels.

Chen (1986), Chaaban et al (1986) and Parker and Underwood (1998) all model a bi-linear unloading expressed as a function of ε^p with the point of slope discontinuity defined by the BEF data of Milligan et al (1966), however these models do not appear capable of further extension to true non-linear unloading. Whilst Jahed and Dubey (1997) used the same bilinear unloading model as Parker and Underwood (1998), their method has the potential to accept fully varying data of the form presented in equation (1). This extension is demonstrated in the next section.

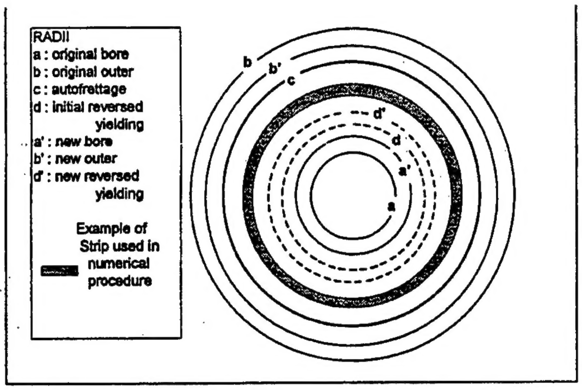

Figure 2 : Thick Cylinder Geometry

AUTOFRETTAGE RESIDUAL STRESSES - CURRENT WORK

The method is formulated in Jahed and Dubey (1997). There are two typographical errors in equation (17) of that paper, however these do not affect the form of the formulation employed herein. Summarizing the overall procedure:

a. The tube to be autofrettaged is divided into a number of thin strips for numerical analysis, Figure 2. Each strip is in the form of a thin tube and is obliged (via the formulation) to satisfy requirements of equilibrium and compatibility at its inner and outer interfaces with adjacent strips and to conform to a particular physical law with axi-symmetric restrictions (namely Lamé's equations, see Chakrabarty (1987)). Lamé's equations, (2) and (3), provide the hoop stress, σ_θ and the radial stress σ_r at radius r in a thick cylinder, internal radius a, external radius b when subjected to internal pressure p.

$$\sigma_\theta = \frac{a^2 p}{(b^2-a^2)}\left[1+\frac{b^2}{r^2}\right] \qquad (2)$$

$$\sigma_r = \frac{a^2 p}{(b^2-a^2)}\left[1-\frac{b^2}{r^2}\right] \qquad (3)$$

Note that whilst Jahed and Dubey (1997) employ strips of constant radial thickness this is not necessary to the basic formulation; indeed the current work indicates clear advantages in ensuring thinner strips in regions of particular sensitivity, relating to the rate of change of material properties (normally near the bore) and/or convergence requirements (normally near the elastic-plastic interface).

b. Numerical autofrettage pressurization modeling begins with an initial quasi-elastic, or hypothetical, stress distribution (normally the Lamé solution for the particular bore pressure). The Tresca or Von Mises equivalent stress, Chakrabarty (1987), for each strip is then forced to follow the uniaxial loading profile (A-B-Cn) by ensuring that each strip which remains truly elastic during loading ($b \le r \le c$) has elastic material properties E and ν (Poisson's ratio), whereas strips ($a < r < b$) have quasi-elastic (effective) properties E_{eff} and ν_{eff}. The projection method of defining an initial and updated value of E_{eff} for the center of each strip is defined and illustrated diagramatically in Jahed and Dubey (1997). Once E_{eff} has been determined ν_{eff} is immediately obtained from:

$$\nu_{eff}=[2\nu+(E/E_{eff}-1)]/[2+2(E/E_{eff}-1)] \qquad (4)$$

Note that this represents an equivalent but simpler form of equation (13) in Jahed and Dubey (1997); furthermore it lends itself to situations in which materials data are available only in discrete form and require interpolation.

An iterative procedure is employed to refine E_{eff} for each strip. With each iteration the previous E_{eff} and ν_{eff} is replaced by the revised figures. Convergence using the Jahed and Dubey (1997) 'projection' method is monotonic, reasonably rapid and easily assessed.

In order to make subsequent comparisons more straightforward a standard tube geometry and bore pressurization loading is defined as follows:

STANDARD GEOMETRY:
initial outer radius, b : 100mm
initial inner radius, a : 50mm
STANDARD CYCLIC PRESSURIZATION:
bore pressure, $p = \sigma_Y/3$, $E/\sigma_Y = 188.2$ (typical for gun tubes).
where σ_Y is uniaxial yield strength of the material
OTHER GEOMETRICAL DEFINITIONS:
maximum radius of initial yielding during autofrettage, c
maximum radius of reversed yielding on initial unloading, d
increased inner radius after material removal, a'
reduced outer radius after material removal, b'
maximum radius of reversed yielding after material removal, d'

Figure 3 shows a typical set of results for the stress state at the peak of an autofrettage loading cycle. These results are for the standard geometry with Tresca criterion under plane stress and 60% overstrain (overstrain is defined as the proportion of the wall thickness of the tube in which yielding occurs during the autofrettage loading cycle). Percentage prior plastic strain, ε^p is noted at the peak of autofrettage loading for the mid-point of each strip. The loading is assumed to be elastic - perfectly plastic, a good approximation in typical gun steels. However any loading profile e.g. linear strain-hardening, Ramberg-Osgood (Chakrabarty (1987)) may be easily incorporated if desired. Any combination of Tresca or Von Mises and plane stress or plane strain is covered by the formulation. If engineering plane strain conditions are to be satisfied (i.e. zero net force exerted by total axial stresses) a further iteration is required. In practice it was observed that

the difference between the hoop and radial stress predictions for a given level of overstrain based upon plane stress and plane strain was minimal (less than 0.5%).

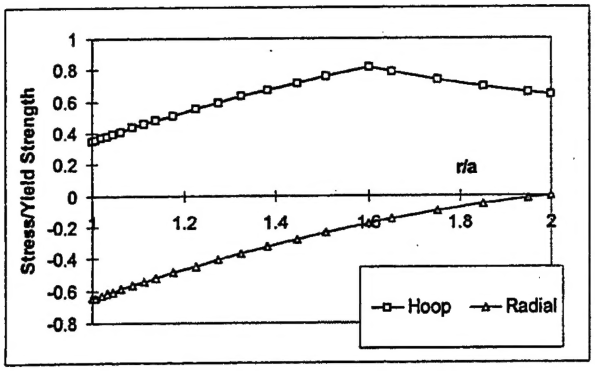

Figure 3 : Stress State at Peak of Autofrettage Loading, b/a=2, 60% Overstrain, Tresca and Plane Stress Conditions Assumed

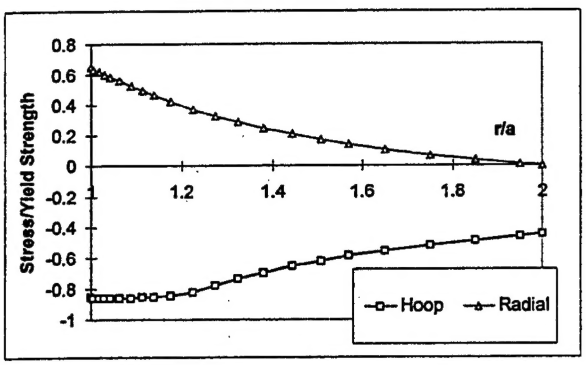

Figure 4 : Stresses Produced During Unloading Incorporating Bauschinger Effect, b/a=2, 60% Overstrain. Tresca and Plane Stress Conditions Assumed

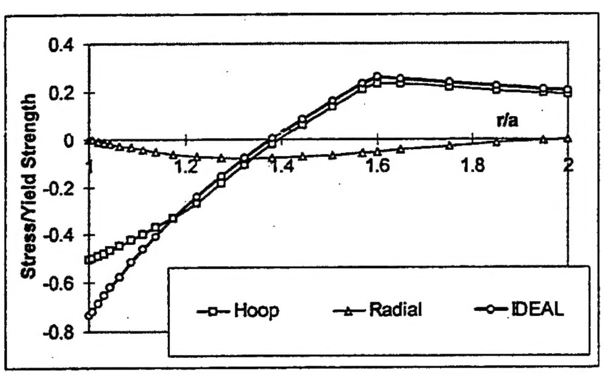

Figure 5 : Total Residual Stress Profile, b/a=2, 60% Overstrain. Tresca and Plane Stress Conditions Assumed

Incorporating the unloading data based upon the calculated prior plastic strain values, equation (1), into the above numerical formulation provides subsequent results. Figure 4 shows the stresses produced during the reversed-loading cycle for the tube considered above. Finally Figure 5 shows, by summation of the profiles in Figures 3 and 4, total residual stress profiles compared with the usual 'ideal' perfectly elastic unloading assumption, Chakrabarty (1987).

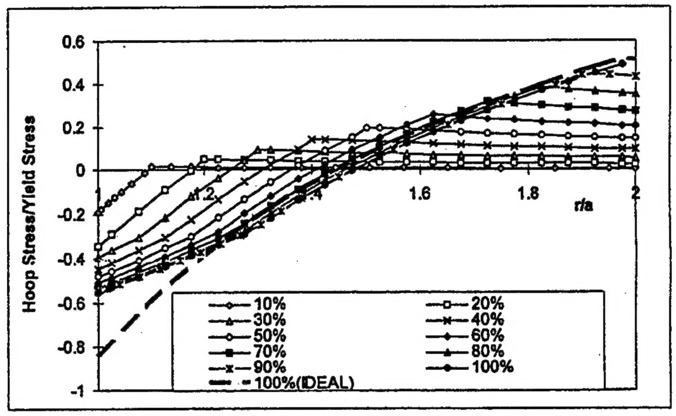

Figure 6 : Total Hoop Residual Stress Profile for b/a=2 with Various Percentage Overstrains. Tresca and Plane Stress Conditions Assumed

Results of a series of analyses for various overstrains between 10% and 100% are presented in Figure 6. Both 'ideal' and Bauschinger effect profiles are presented. Figure 7 shows the values of actual and ideal hoop stress at the bore. Noteworthy effects from Figures 6 and 7 include:

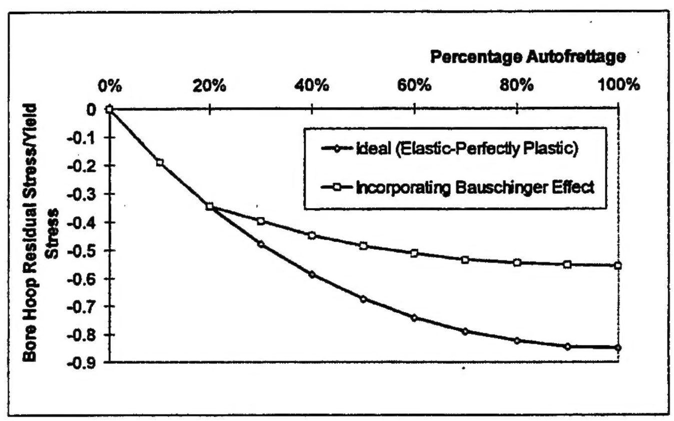

Figure 7 : Bore Hoop Residual Stress versus Percentage Overstrain for Ideal and Bauschinger Distributions. b/a=2, Tresca and Plane Stress Conditions Assumed

- A large reduction in bore hoop stresses as a result of Bauschinger effect.

- The Bauschinger effect penetrates much deeper into the tube than previous attempts at modeling typical gun steels have suggested: approximately 22% and 30% of wall thickness for overstrains of 60% and 100% respectively for a tube of radius ratio 2.0. Previous work, Chen (1986), Parker and Underwood (1998) has suggested equivalent depths of around 13% and 18% respectively. Likewise Chaaban et al (1986) who analyzed 100% overstrain report an equivalent depth of 17% of wall thickness.
- A minimum value of hoop stress at the bore associated with a 'saturation' value of $\varepsilon^p = 2\%$. This is a direct result of the constant BEF values observed by Milligan et al (1966) for $\varepsilon^p \geq 2\%$.
- Very limited benefit (in terms of increased compressive hoop stresses in the near-bore region) as a result of overstrain above 60%.

PRESSURE FOR FURTHER YIELDING ON RE-PRESSURIZATION

Kendall (1987) compared several of the models then available to predicting re-yielding pressures for autofrettaged tubes. By employing results presented in this paper it is possible to address the re-yielding phenomenon.

Assuming that the tensile yield strength is unaffected by prior loading and that the elastic modulus is unchanged it is a straightforward procedure to superpose elastically upon the autofrettaged tube an increasing bore pressure until the point at which the selected yield criterion is reached.

If the tube is re-pressurized after initial autofrettage re-yielding will occur first at the bore. The pressure at which this occurs is designated p**. Expressed in terms of Tresca's yield criterion this occurs when

$$(\sigma_\theta^{p^{**}} + \sigma_\theta^A) - \sigma_r^{p^{**}} = \sigma_Y \qquad (5)$$

or

$$(\sigma_\theta^{p^{**}} - \sigma_r^{p^{**}}) = \sigma_Y - \sigma_\theta^A \qquad (6)$$

where σ_θ^A is the bore hoop residual stress following initial autofrettage, presented in Figure 7.

Combining equation (6) with (2) and (3) provides:

$$p^{**} = \left(\frac{b^2 - a^2}{2b^2}\right)(\sigma_Y - \sigma_\theta^A) \qquad (7)$$

which reduces, in the case of zero residual stress, to the familiar expression for the pressure for initial yielding of a thick cylinder, Chakrabarty (1987).

Figure 8 shows the results of such a process for the standard geometry over the full range of overstrains from 0 to 100%. For comparison the range of pressures for initial autofrettage is also shown.

INFLUENCE OF BAUSCHINGER EFFECT UPON FATIGUE LIFETIME

It is possible to make some rapid approximate predictions of the impact of Bauschinger effect upon fatigue lifetimes for cyclically pressurized, autofrettaged tubes containing pre-existing crack-like defects. Assuming a tube of standard geometry and pressurization the positive cyclic range of hoop stress at the bore is:

$$5/3.Y/3 + \sigma_\theta^A \qquad (8)$$

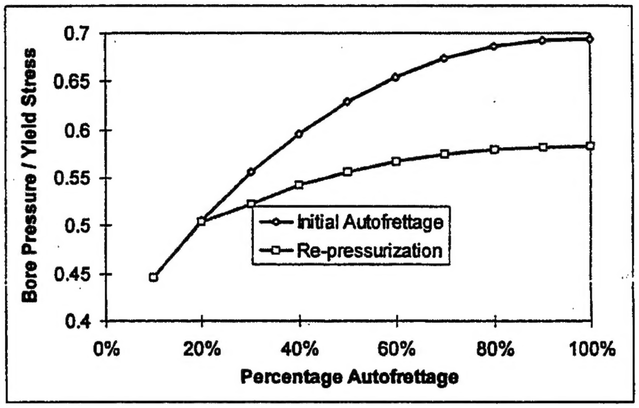

Figure 8 : Pressure for Yield of Autofrettaged Tube on Re-pressurization, b/a=2

where the 5/3 term comes from Lamé's equations (2) and (3) for the hoop stress at the bore of a pressurized tube and σ_θ^A (< 0) is the value of bore hoop stress arising from the autofrettage process.

Approximate fatigue lifetime ratios (Actual Lifetime with Bauschinger Effect/Ideal Lifetime) are then assessed on the basis of:

$$Lifetime\_Ratio = ([5/3.\sigma_Y/3 + \sigma_\theta^A + \sigma_Y/3] / [5/3.\sigma_Y/3 + \sigma_\theta^{IDEAL} + \sigma_Y/3])^3 \qquad (9)$$

Here IDEAL refers to the residual hoop stress obtained from the ideal (i.e. BEF = 1) solution. The $\sigma_Y/3$ terms arise because bore pressure infiltrates and acts upon the crack surface and the exponent 3 is typical of the exponent in the Paris and Erdogan (1963) fatigue crack growth law.

Results of this calculation of lifetime ratio based upon the bore residual stresses shown in Figure 7 are presented in Figure 9. Similarly Figure 10 shows the ratios Actual Lifetime/Lifetime with zero autofrettage and Ideal Lifetime/Lifetime with zero autofrettage.

It is also possible to undertake a more sophisticated prediction of lifetimes, Parker and Underwood (1998). Fatigue lifetimes, based upon stress intensity factor solutions of extremely high accuracy (errors < 0.5%) determined by the Modified Mapping Collocation technique, Andrasic and Parker (1984), and packaged as weight function data, Andrasic and Parker (1982), are presented in Figure 11. The calculations were based upon the following geometrical and materials properties: a = 50 mm; b = 100 mm; four initial, equally spaced, straight-fronted radial bore cracks of length 0.5 mm; internal cyclic pressure 400 MPa; Young's modulus, E, 200 GPa; Yield

Strength 1200 MPa; Paris Law coefficient, C, 6.52E-12; Paris Law exponent, m = 3.

The stress intensity factor calculations take full account of thru-the-thickness variation of residual and pressurization stresses. Overstrains from 0 to 100% were examined, and lifetimes calculated for the cases of ideal autofrettage (based on Tresca criterion) and incorporating Bauschinger effect. Clearly Figure 11 shows a very similar behavior to that derived from the simpler formulation and presented in Figure 10.

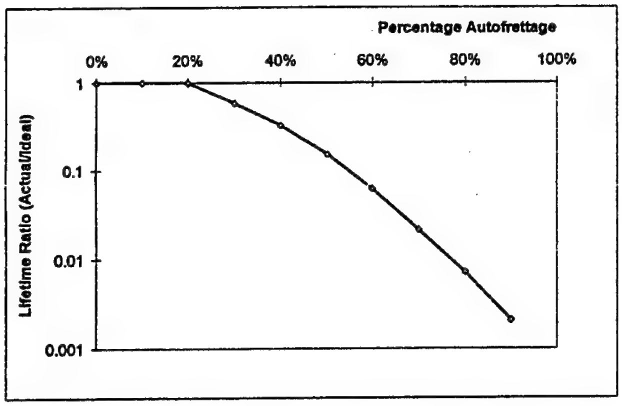

Figure 9 : Approximate Fatigue Lifetime Ratio, Actual Lifetime/Ideal Lifetime

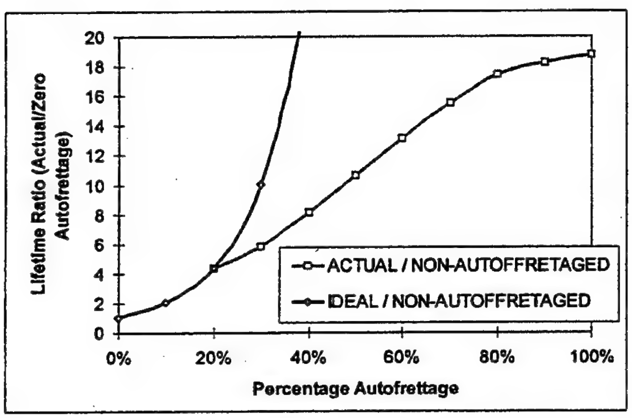

Figure 10 : Approximate Fatigue Lifetime Ratios, Actual Lifetime/Lifetime Without Autofrettage and Ideal Lifetime//Lifetime Without Autofrettage

REMOVAL OF MATERIAL FROM ID AND/OR OD

It is frequently the case that material is removed from the ID and/or OD of a tube following autofrettage. There are four possible scenarios:

Removal of Material from OD

If material is removed from the OD (whether or not material has previously been removed from the bore) there can be no further yielding as a result of the OD removal and only a straightforward superposition procedure is required. The sequence is:

a. Calculate full distribution of stresses within the original tube inner radius a and outer radius b with autofrettage radius c. Note radial stress at b', $\sigma_r^{b'}$, where b' is to be the outside radius after material removal.

b. Calculate the stress distribution for a tube inside radius a, outside radius b', with $(-\sigma_r^{b'})$ applied at b'.

c. Superpose (add) the stress field from (a) to that from (b).

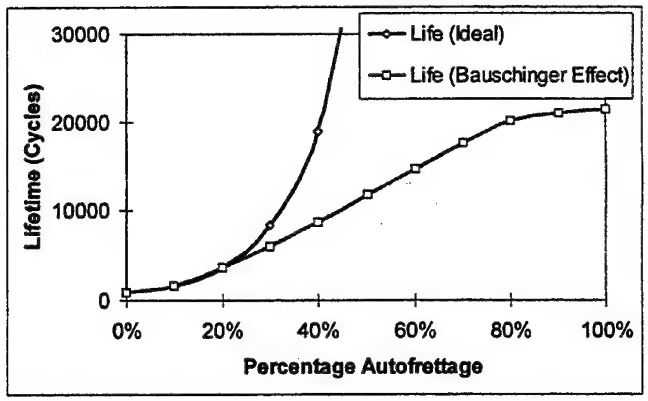

Figure 11 : Predicted Lifetimes as a Function of Percentage Autofrettage based upon Accurate Stress Intensity Data and Paris' Law (b/a=2)

Removal of Material from ID - without Bauschinger Effect

If the material did not exhibit any Bauschinger effect on unloading the modeling of material removal from the bore would be the elastic analogy of the procedure outlined above for removal of material from OD. Namely:

a. Calculate full distribution of stresses within the original tube inner radius a and outer radius b with autofrettage radius c. Note radial stress at a' , $\sigma_r^{a'}$, where a' is to be the new inner radius after material removal.

b. Calculate the stress distribution for a tube inside radius a', outside radius b, with $(-\sigma_r^{a'})$ applied at a'.

c. Superpose (add) the stress field from (a) to that from (b).

Such elastic procedures lead to the familiar conclusion for the non-Bauschinger case that autofrettage out to radius c of a tube inner radius a, outer radius b ($b \leq 2.22$) followed by subsequent removal of material from ID (to a') and/or from OD (to b') produces a stress distribution essentially identical to that which would have been achieved by autofrettage out to radius c of a tube inner radius a', outer radius b'.

Removal of Material from ID - with Bauschinger Effect

This case is more complex since there is the likelihood of further inelastic behavior near the bore. Several steps are necessary within the numerical procedure to ensure that such inelasticity is properly represented.

a. Determine the full stress field at the peak of the autofrettage loading cycle (bore pressure p*) in the original tube, inner radius a, outer radius b with autofrettage radius c. Note the radial pressure at radius a', $p_L^{a'}$.

b. Determine the residual stress field after removal of p* using the elastic-plastic procedure previously described. Note (for subsequent use) the radial pressure at radius a', $p_U^{a'}$.

c. Determine the full stress field at the peak of the autofrettage pressure cycle (bore pressure $p_L^{a'}$) in the reduced tube, inner radius a', outer radius b.

d. Determine the residual stress field in the reduced tube after removal of $p_L^{a'}$ using the procedure previously described.

The stress field (b) thus properly represents the residual stress field which would be generated in the original tube whilst (d) properly represents the residual stress field which would be generated after the original tube subsequently had material removed from the bore.

Removal of Material from OD Followed By Removal of Material from ID

In this case there may or may not be further inelastic behavior at the bore. To assess whether further inelasticity occurs:

a. Determine the residual stress field after removal of p* using the elastic-plastic procedure previously described. Note (for subsequent use) the radial pressure at radius a', $p_U^{a'}$ and at radius b', $p_U^{b'}$.

b. If $p_U^{b'} \geq p_U^{a'}$ comparison of the additional deviatoric stress (Tresca criterion) indicates that no further inelastic behavior occurs and the analysis is a simple superposition process. However if $p_U^{b'} < p_U^{a'}$ further yielding will occur and the stress field must then be determined by calculating iteratively the autofrettage pressure required to cause yielding out to radius c in a tube of inner radius a' and outer radius b' by the method developed herein.

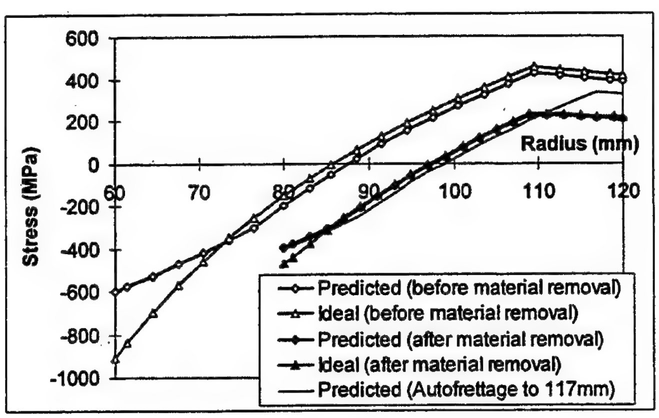

Figure 12 : Hoop Stresses Before and After Removal of Significant Amount of Material from the bore of an Autofrettaged Tube

CASE STUDY

As an example consider the case of a thick cylinder, inner radius 60mm outer radius 120mm which is autofrettaged to 80% overstrain and is then bored out along a proportion of its length to a new bore radius of 80mm.

The results of the procedure outlined above for a steel of yield strength 1104 MPa following the unloading behavior of equation (1) are shown in Figure 12. Note the significant reduction in the magnitude of residual stress at the bore of the original (thicker) tube. However it is also significant, and perhaps surprising, that after material removal there is still a significant loss of compressive bore hoop stress associated with the Bauschinger effect. Indeed the Bauschinger affected zone (BAF) which extended to within 43mm of the OD before removal is now only 32.5mm from the OD.

Now consider potential improvements by extending the depth of the autofrettage. If the autofrettage radius were increased to 117mm the residual stress in the thinner-walled section would be modified as shown in Figure 12. Examination of the minute increase in near-bore compressive residual stress indicates that neither lifetime nor pressure for re-yielding would be improved by such action.

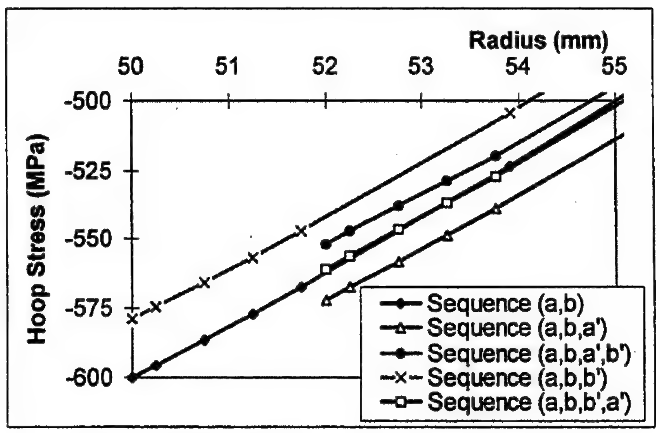

Figure 13 : Effect of Sequence of Material Removal - 2mm from OD and 2mm from ID. Various sequences.

REMOVAL OF MATERIAL FROM BOTH ID AND OD

The fact that different methods of analysis may be required for ID as opposed to OD material removal implies an important manufacturing concept. *If there is a choice in the sequence of material removal from autofrettaged tubes such material should always be removed from OD before ID.*

As an example consider a tube of the standard geometry having yield strength 1104 MPa which is autofrettaged to 80% overstrain. The tube then has 2mm of material removed from both ID and OD. The impact of the sequence of material removal is shown in Figure 13 for the important near-bore region. The sequence notation is intuitive, e.g. sequence a, b, b', a' indicates original autofrettaged tube, inner radius a, outer radius b, subsequently had its outer radius reduced to b' and subsequently had its inner radius reduced to a'.

Clearly, although the final tube dimensions are the same, there is a greater residual compressive stress at the bore after sequence (a, b, b', a') than after sequence (a, b, a', b'). Employing once again an expression similar to equation (9) this differential may be expressed as a ratio of fatigue lifetimes. Assuming the previous ratio of 3 for cyclic pressure to yield strength this calculation gives a lifetime ratio of 0.94 i.e. a reduction in lifetime of some 6% arising simply from the selection of an incorrect order of machining.

SACHS' METHOD

Sachs' experimental method, Weiss (1956), Davidson et al (1963), involves attaching strain gauges to the OD of an autofrettaged cylinder and then incrementally machining material from the bore, hence increasing bore diameter in a series of steps. Before and after each step OD axial and hoop strains are measured and the differences are then used to compute radial stress at the prospective radius *prior to* material removal. This figure is then used to establish the associated residual hoop stress at the ID prior to material removal. Critically the method assumes that the material behaves elastically during and after material removal and that OD strains can thus be related to ID stresses via Lamé's equations.

To test this hypothesis of elastic behavior the current non-linearnumerical model is used to simulate the Sachs experimental process. A tube of standard geometry having 80% autofrettage was modeled. The initial residual stress field incorporating Bauschinger effect was modeled as was the subsequent incremental process of removing material from the bore (see earlier section Removal of Material from ID - with Bauschinger Effect). The initial residual radial stress at each incremental bore radius was used to predict changes in OD hoop stress due to simulated elastic unloading. Such changes were also predicted via the numerical elastic-plastic procedure.

The results are presented in Figure 14. They clearly indicate significant overestimates in residual hoop stress evaluated via the Sachs' procedure. The Sachs' overestimates are particularly large in the near-bore region and would produce enormous errors if used to calculate fatigue lifetime via equation (9).

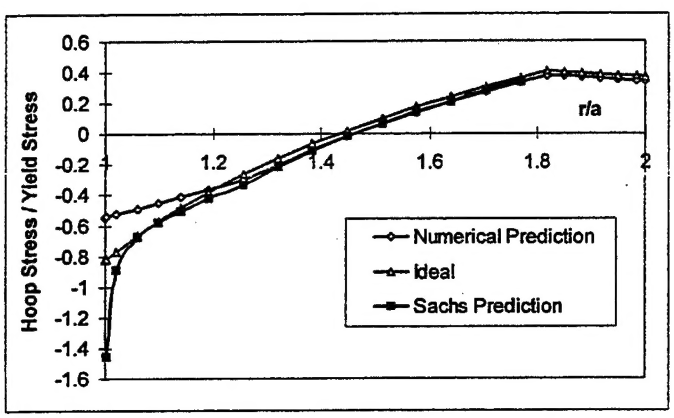

Figure 14 : Overestimate in Sachs' Residual Stress Predictions Arising from Bauschinger Effect

The above procedure was used to predict the outcome of the Sachs' procedure for one set of experimental results reported in Davidson et al (1963); Figure 15 shows the comparison with data recovered from Figure 11 of that paper. This case involved a tube with a radius ratio of 2.3 and bore diameter of 19.05mm which was swage-autofrettaged to 54% overstrain. The yield strength of this particular tube was 882 MPa.

The proximity of the expected and actual Sachs predictions shown in Figure 15 gives considerable confidence in the current numerical procedure.

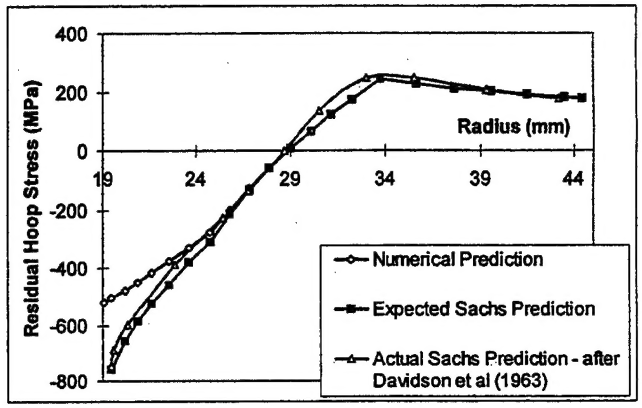

Figure 15 : Overestimate in Sachs Residual Stress Predictions Arising from Bauschinger Effect - Comparison with Experimental results

CONCLUSIONS AND RECOMMENDATIONS

The proposed methods for prediction of residual stresses and lifetimes for a Bauschinger-affected autofrettaged tube (BAAT) provides a straightforward design procedure. It has been shown that the Bauschinger effect produces a large reduction in bore hoop stress. The Bauschinger effect penetrates much deeper into the tube than previous attempts at modeling typical gun steels have suggested: approximately 22% and 30% of wall thickness for overstrains of 60% and 100% respectively for a tube of radius ratio 2.0.

When such a BAAT is re-pressurized there will be re-yielding below the original autofrettage pressure in cases where autofrettage radius exceeds approximately 1.2 x bore radius. For typical levels of overstrain re-yielding occurs when the bore pressure reaches approximately 60% of the original autofrettage pressure.

After a tube has been autofrettaged to some typical overstrain (say 1.8 x bore radius) there are small increases in compressive bore hoop stress arising from further autofrettage whereas there is a significant increase in OD tensile hoop stress with its attendant risks.

The predicted residual bore hoop stresses may be employed in the approximate determination of fatigue lifetimes for thick cylinders with pre-existing cracks. These calculations employ the ratio of the range of positive hoop stresses. Such calculations indicate orders of magnitude reduction in lifetimes of cyclically pressurized BAATs as compared to

non-BAATs. The lifetime ratio for a BAAT of radius ratio 2, as compared to a non-autofrettaged tube, is shown to be in the range 14 - 18 for typical overstrain levels. A parallel, more extensive, procedure using accurate stress intensity calculations and a fatigue crack growth law shows very similar behavior.

Removal of material from a BAAT may result in further yielding of the tube and, in general, requires further non-linear analysis. The procedure for a range of material removal scenarios is detailed in this paper. It was shown that, where material is to be removed from both the ID and the OD of a BAAT, the removal sequence should be OD removal followed by ID removal in order to maximize compressive bore hoop stress and associated lifetimes.

The Sachs' experimental method for determination of residual stress in autofrettaged thick cylinders was critically examined. The Sachs' procedure assumes elastic behavior during incremental removal of bore material. It was demonstrated that such an assumption will lead to a very large overestimate of residual hoop stress near the bore. A specific example, based upon earlier experimental work, was used to demonstrate the ability of the non-linear model to predict the outcome of one such Sachs' experimental analysis.

There were two implicit assumptions in the approach employed herein. Firstly, that a uniaxial tension-compression test is capable of providing sufficient data to perform a subsequent two, or three, dimensional stress analysis. Secondly that, in the case of initial re-pressurization to produce further yielding, cyclic effects are relatively insignificant. Each of these assumptions requires further experimental verification.

ACKNOWLEDGMENTS

Much of this work was undertaken during an attachment by one of the authors (APP) to the US Army Armament Research, Development and Engineering Center (ARDEC), Watervliet, NY. The attachment was arranged via the European Research Office of the US Army Research, Development and Standardization Group (UK).

The authors gratefully acknowledge valuable discussions with Dr. R. N. Dubey of the University of Waterloo, Ontario, Canada.

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